1.0 The Direct and Inverse Piezoelectric Effect

In 1880, while performing experiments with tourmaline, quartz, topaz, cane sugar and Rochelle salt crystals, Pierre and Jacques Curie discovered that when mechanical stress was applied to a crystal, faint electric charges developed on the surface of that crystal. The prefix “piezo” comes from the Greek piezein, which means to squeeze or press. As a result, piezoelectricity is electrical charge that is produced on certain materials when that material is subjected to an applied mechanical stress or pressure. This is known as the direct piezoelectric effect.

The converse or inverse piezoelectric effect, or the application of an electric field to induce strain, was discovered using thermodynamic principles in 1881 by Gabriel Lippmann. It is the inverse piezoelectric effect that enables piezoelectric materials to be used in positioning applications.

2.0 Piezo Actuator Materials

Although many materials exhibit the inverse piezoelectric effect, the most popular and widely applicable piezoelectric material by far is PZT, or lead-zirconium-titanate. The term PZT is generally used to refer to a wide range of ceramics which display different properties depending on the grain size and mixture ratios of their main raw materials: lead, zirconium and titanium. The properties of the ceramic can also be manipulated by adding dopants and making adjustments to the manufacturing process. The recipes for particular materials are usually proprietary and vary between suppliers.

2.1 RoHS Exemption

Despite the presence of lead as a doping material, PZTs are exempt from RoHS directive 2002/95/EC due to a lack of a suitable replacement material. Although efforts are underway to develop alternative materials, no suitable alternative is expected in the field for years to come.

3.0 Properties of Piezo Actuators

3.1 Displacement Performance

The response of a piezoelectric material to an applied stress or applied electric field depends on the direction of application relative to the polarization direction. Because of this, most electrical and mechanical properties that describe piezo materials are direction dependent, as well.

The inverse piezoelectric effect can be described mathematically as:

$$x_j = d_{ij} \cdot E_i \quad (Eq. 1)$$

where, $x_j$ is strain (m/m), $d_{ij}$ is the piezoelectric charge coefficient (m/V) and is a material property, and $E_i$ is the applied electric field (V/m). The subscripts $i$ and $j$ represent the strain direction and applied electric field direction, respectively. Electric field is a voltage across a distance, so large electric fields can be generated with small voltages if the charge separation distance is very small.

As a general rule, the strain ($x_j$) for most PZT materials found on the market is around 0.1 to 0.15% for applied electric fields on the order of 2 kV/mm. For example, a 20 mm long active-length PZT actuator will generate approximately 20-30 μm of maximum displacement. One can easily see that to generate 250 μm, a PZT stack would be approximately 170 to 250 mm long. Therefore, most piezo flexure stages with >50 μm of travel use lever amplification to achieve longer travels in a more compact package size. A tradeoff is made between the final device package size and stiffness because the stiffness of the device decreases with the square of the lever amplification ratio used. Aerotech’s piezo nanopositioning stages are optimized to provide superior mechanical performance in a compact stage package.

3.2 Hysteresis Effects

Piezoelectric materials are a subset of a larger class of materials known as ferroelectrics. Ferroelectricity is a property of certain materials that have a spontaneous electric polarization that can be reversed by the application of an electric field. Like the magnetic equivalent (ferromagnetic materials), ferroelectric materials exhibit hysteresis loops based on the applied electric field and the history of that applied electric field. Figure 1 shows an illustration of a strain ($X$) versus electric field ($E$) “butterfly” curve for a PZT material driven to its excitation limits.

Figure 1. Strain (displacement) behavior of a ferroelectric material like PZT with an applied electric field driven to its excitation limits.
As the electric field is cycled from positive to negative to positive, the following transformations occur in the piezo actuator:

A: Initially, strain increases with electric field and is only slightly nonlinear. As the electric field is increased, the dipoles of all the grains will eventually align to the electric field as optimally as is possible and the distortion of the grains will approach a physical limit.

B: When the field is reversed, strain decreases more slowly due to the reoriented dipoles. As the field gets smaller, the dipoles relax into less ideal orientations and strain decreases at a faster rate.

C: As the field becomes negative the dipoles are forced away from their original orientation. At a critical point they completely reverse direction and the piezo actuator becomes polarized in the opposite direction. The electric field at the point of polarization reversal is known as the coercive field ($E_c$).

D: After polarization reversal, the piezo expands again until it reaches its physical strain limit.

E: The electric field is reversed again and the same hysteretic behavior that occurred along curve B occurs as strain decreases.

F: The electric field is driven to the coercive limit for the opposite polarization direction and the dipoles reorient to their original polarization.

G: The piezo actuator expands with the applied electric field to its physical limit.

For positioning applications, piezo actuators are generally operated with a semi-bipolar voltage over an area of the curve (ABC) away from the saturation and coercive field limits. An example of displacement versus applied voltage for a piezo actuator stack in this region of the curve is shown in Figure 2.

Aerotech amplifiers take full advantage of the semi-bipolar operation of piezoelectric stack actuators. Our actuators are designed to operate from -30 V to +150 V with very high voltage resolutions. Over this voltage range, open-loop hysteresis values can be as large as 10-15% of the overall open-loop travel of the piezo stage. Operation of the piezo stage in closed-loop effectively eliminates hysteresis of the actuator enabling positioning repeatabilities in the single-digit nanometer range.

3.3 Creep and Drift

The response time for a piezo ceramic subjected to an electric field is much faster than the reorientation time of the individual dipoles. This phenomenon causes undesirable behavior in open-loop position control. When an electric field is applied, the piezo stack will make a corresponding displacement almost instantaneously. If the field is then held constant, the piezo stack will continue to move slowly as the dipoles reorient, a phenomenon known as creep. It can take many minutes or even hours to reach steady state, with strain increasing by as much as 1% to 5% past the initial strained position. There is a similar effect called zero point drift. When an electric field is removed, the dipoles will gradually relax and motion will continue slowly until a steady state is reached. Operating the piezo actuator or stage in closed-loop control eliminates this drift because the controller is compensating for this movement in real time to keep the output motion at the desired position.

3.4 Force and Displacement

3.4.1 Force Versus Displacement Characteristics

The force generated by an actuator acting in the polarization direction is completely independent of the overall length of the actuator and is only a function of the cross-sectional area of the actuator and the applied electric field. An illustration of the force versus displacement output of a piezoelectric actuator with various applied voltages is shown in Figure 3.

Figure 2. Typical hysteresis curve of a piezoelectric stack actuator operating from -30 V to +150 V.

Figure 3. Force vs. displacement output of a piezo actuator at various applied voltages.
A few interesting characteristics become evident upon inspection of Figure 3. The piezo actuator’s force and displacement increase as the applied voltage is increased. The maximum force output of a piezo actuator, or blocking force, occurs when the rated voltage is applied across the actuator and the output of the actuator is “blocked” or not allowed to move. As the actuator expands, the force production capability reduces until the force output reaches zero at the maximum rated displacement of the actuator.

### 3.4.2 Displacement With a Constant External Load

Figure 4 illustrates the case of a piezo actuator or stage with a constant, external load applied.

In the case of a piezo stage or actuator without any applied load (case 1), the stroke of the piezo is given as \( \Delta L_1 \). When a mass is applied to the piezo stage (with expansion in the direction of gravity), the initial deflection \( \Delta L_0 \) is calculated as:

\[
\Delta L_0 = \frac{F}{k_p} = \frac{m \cdot g}{k_p} \quad (Eq. 2)
\]

where \( k_p \) is the stiffness of the piezo stage in the direction of motion and \( m \) is the applied mass. With mass \( m \) applied to the piezo stage, the stage is compressed a distance \( \Delta L_0 \) but the stroke \( \Delta L_2 \) remains the same as the unloaded stage. That is:

\[
\Delta L_2 = L_1 \quad \text{(constant, external load)} \quad (Eq. 3)
\]

### 3.4.3 Displacement With an External Spring Load

Figure 5 illustrates a case where a piezo actuator or stage is driving against an external spring load.

In the case of a piezo stage or actuator without any applied load (case 1), the stroke of the piezo is given as \( \Delta L_1 \). For case 2 when driving against a spring load, the piezo stage stiffness \( (k_p) \) and the external stiffness \( (k_e) \) act in series and decrease the overall stroke of the actuator. The stroke in case 2 is given by:

\[
\Delta L_2 = \frac{L_1 \cdot k_p}{(k_p + k_e)} \quad (Eq. 4)
\]

It is evident upon inspection of Equation 4 that in order to maximize the stroke of the piezo stage, the piezo stage stiffness \( (k_p) \) should be much larger than the external spring stiffness \( (k_e) \).

### 3.5 Capacitance

PZT actuators can be modeled electrically as a capacitor. The principle equation that describes a capacitor in terms of geometry and material properties is:

\[
C = \varepsilon \cdot \frac{A}{T} \quad (Eq. 5)
\]

where \( C \) is capacitance (units of F), \( A \) is the cross-sectional area of the capacitor perpendicular to the direction of the electric field (units of m²), \( T \) is the thickness of the dielectric material separating the charge (units of m), and \( \varepsilon \) is the material permittivity of the dielectric material separating the charge. The material permittivity is described as:

\[
\varepsilon = \varepsilon_r \cdot \varepsilon_0 \quad (Eq. 6)
\]

where \( \varepsilon_0 \) is the permittivity of a vacuum (~8.85 x 10⁻¹² F/m), and \( \varepsilon_r \) is the relative permittivity of the material (also called the dielectric constant).

Low-voltage, multi-layer actuators are generally used for nanopositioning because they allow for 0.1% to 0.15% nominal strains with low voltages (<200 V). The maximum applied electric field across these actuators are in the range of 1-4 kV/mm. Because these actuators are constructed...
from thin layers (typically 50 to 200 μm thick) separated by electrodes, the resulting applied voltages are lower (<200 V) compared to high-voltage actuators (~1000 V) where layer thickness is ~1 mm. The thickness of each layer ($T_{layer}$) can be defined as the overall active length of the piezo actuator ($L_o$) divided by the number of layers (n). The piezo stack capacitance of a multi-layer actuator can then be expressed as a function of the number of layers (n) and the overall active length ($L_o$), as follows:

$$C = n^2 \cdot \varepsilon \cdot \frac{A}{L_o} \quad (Eq. 7)$$

Typical capacitances of low-voltage, multi-layer piezo actuators used in nanopositioning applications are between 0.01 to 40 μF. The capacitances specified in Aerotech data sheets are measured at small signal conditions (1 Vrms at 1 kHz). For larger signal operation (100-150 V), an increase in capacitance by as much as 60% should be expected. This capacitance increase should be used when performing sizing calculations (see Section 5).

The current ($i$) flowing through a capacitor ($C$) is proportional to the change in voltage with respect to time. This is mathematically represented as:

$$i = C \cdot \frac{dV}{dt} \quad (Eq. 8)$$

This simple relationship will be needed to adequately size amplifiers required to drive piezoelectric stages (see Section 5).

### 3.6 Heating and Power Dissipation

An ideal capacitor does not dissipate any power in terms of heat. However, in practice a piezo actuator does not act as an ideal capacitor and does have some internal resistance that generates heat when current is flowing through the actuator. The dielectric loss factor, or loss tangent, is defined as:

$$\tan \delta = \frac{ESR}{X_c} \quad (Eq. 9)$$

where ESR is the equivalent series resistance of the capacitor and $X_c$ is the capacitive reactance. The loss tangent can also be written as the ratio of active (resistive) power ($P$) to reactive power ($Q$):

$$\tan \delta = \frac{P}{Q} \quad (Eq. 10)$$

The higher the loss tangent, the more energy is converted to heat (energy lost) as an alternating electric field is introduced to the material. For soft PZT materials, which are typically used for nanopositioning applications, the loss tangent generally is between .01 to .03 for lower amplitude signals (~1-10 volts) and can be as high as 0.1 to 0.25 for higher amplitude signals (~50-100 volts).

The reactive power ($Q$) is defined as:

$$Q = \frac{V_{rms}^2}{X_c} \quad (Eq. 11)$$

For a single frequency ($f$) the capacitive reactance is:

$$X_c = \frac{1}{(2 \cdot \pi \cdot f \cdot C)} \quad (Eq. 12)$$

Using Equations 10, 11 and 12, it can be shown that the power dissipated in a piezo actuator for a sinusoidal voltage with an amplitude of $V_{rms}/2$ and frequency $f$ is:

$$P \approx \frac{\pi}{4} \cdot \tan \delta \cdot f \cdot C \cdot V_{pp}^2 \quad (Eq. 13)$$

Equation 13 is a very useful approximation and shows the effects of power loss in piezoelectric devices. This power loss is linearly proportional to the frequency of operation and the capacitive reactance of the piezo actuator, and proportional to the applied time-varying voltage squared. Since voltage is proportional to position, the power loss is proportional to the square of the commanded time-varying position signal applied to the piezo stage.

Figure 6 shows an illustration of how the power loss changes as a function of frequency and applied voltage for a typical piezo actuator with a capacitance of 4 μF.

Temperature rise is proportional to the power dissipated in the actuator. To determine the temperature rise of the piezo actuator or stage requires in-depth knowledge of the exact stage characteristics and design (materials, contact area, etc.). By examining Figure 6, one can see that heating typically only becomes a concern at very large signal amplitudes (e.g., high voltage or large amplitude position) and high frequencies. For most positioning applications, the power dissipation and temperature rise in a piezo nanopositioning stage is negligible. For applications...
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requiring large position oscillations and high frequencies, contact Aerotech’s Application Engineering Department. We will be happy to assist you in sizing the correct piezo nanopositioning device for your exact application.

3.7 Environmental Effects

3.7.1 Humidity
One of the most important factors for ensuring long life is to protect the piezo actuator against humidity. For this reason, Aerotech uses specially-sealed coatings on the actuators that protect the actuator from moisture. Operation at 60% or lower RH environments is preferred as it helps further prolong the life of the actuator.

3.7.2 Temperature
Piezo actuators can be designed to operate at very high temperatures and extremely low temperatures (cryogenic). The extreme upper limit of operation is the Curie temperature of the piezo material. At this temperature, the piezo material loses its piezoelectric effect. Curie temperatures of piezo actuator materials fall between 140°C and 350°C. However, piezoelectric properties are temperature dependent. For this reason, the maximum temperature that Aerotech’s piezo actuators can be used is approximately 80°C. In precision positioning applications, temperatures approaching this can cause serious detrimental effects to the accuracy and performance of the piezo stage.

Piezo actuators are well-suited for operation at extremely low temperatures, as well. The crystals in piezoelectric material remain in their piezoelectric configuration no matter how low the temperature drops. Standard commercially available stack actuators can operate down to -40°C with no problems. The biggest issue in cold environments is not the piezo itself, but induced stress from thermally contracting mechanisms. For extremely cold environments, special design considerations are required for the actuator to survive the cooling process. Carefully chosen electrodes and extremely homogeneous ceramic must be used to prevent cracking because of unmatched thermal expansion coefficients.

Piezo ceramics do operate differently at low temperatures. At these low temperatures, the ceramic stiffens, which causes a decrease in the amount of strain generated per volt. This is offset by increased electrical stability in the crystal structure, allowing fully bi-polar operation. Other advantages of low temperature operation include lower hysteresis, better linearity, lower capacitance and smaller dielectric loss.

For the highest accuracy, Aerotech recommends operation at or near 20°C because that is the temperature in which the nanopositioning stages are built and calibrated. Contact an Aerotech Applications Engineer if extreme temperature environments are expected in your operation as we will assist you in selecting or customizing the proper piezo positioning stage for the highest level of performance in any environment.

3.7.3 Vacuum
Low-voltage (<200 V) piezo actuators are particularly well-suited for vacuum operation. Piezo actuators do not require lubrication that typically requires great care when selecting for ultra-high vacuum applications. Vacuum pressures from 10 to 10^-3 Torr need to be avoided because the insulation resistance of air dramatically decreases in this range (known as the corona area), thus allowing easier dielectric breakdown. Aerotech’s piezo nanopositioning stages can be prepared for ultra-high vacuum operation.

4. Piezo Stage Properties and Nomenclature
Aerotech’s piezo nanopositioning stage series are designed with the end-user in mind. As a result, it is important that our customers have a thorough understanding of our specifications so that they can best be matched to the application or end-process. The following is a description of the specifications and nomenclature used in our data sheets.

4.1 Accuracy/Linearity
As discussed in Section 3.2, piezo actuators exhibit hysteresis and non-linearity when operated in open-loop mode. When operating in closed-loop mode, the non-repeatabilities due to piezo actuator hysteresis are eliminated. However, the piezo stage may still exhibit non-linearities and hysteresis that affect the overall positioning accuracy of the device. The magnitude of these non-linearities are a function of the quality of the closed-loop feedback sensor and electronics used in the design, as well as the quality of the mechanical stage design. With our high-resolution capacitance sensors, advanced electronics and optimized flexure designs, linearity errors below 0.02% are achievable. Accuracy and linearity are measured with precise laser interferometers at a distance of ~15 mm above the moving carriage of the piezo nanopositioner (unless otherwise noted).

The terms accuracy and linearity are sometimes used synonymously when describing the positioning capability of piezoelectric nanopositioners. However, they can have subtle differences in meaning.

Accuracy is defined as the measured peak-peak error (reported in units of micrometers, nanometers, etc.) from the nominal commanded position that results from a positioning stage as it is commanded to move bidirectionally throughout travel.
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Linearity is defined as the maximum deviation from a best-fit line of the position input and position output data. Linearity is reported as a percentage of the measurement range or travel of the positioning stage.

An example of the raw measurement results from an accuracy and linearity test is shown in Figure 7. The accuracy plot is shown in Figure 8. Notice how the accuracy results have a small residual slope remaining in the data. The deviation of a best-fit line to the measurement data taken in Figure 7 is used to calculate the linearity error. The residuals from this best-fit line and an illustration of how linearity error is calculated is shown in Figure 9.

In conclusion, the term accuracy is used to quantify both sensitivity effects (slope of measured versus actual position) as well as nonlinearities in positioning and is reported as a pk-pk value. The term linearity is used to quantify the effects of nonlinearities in positioning only and is reported as a maximum error or deviation of the residuals from the best fit line through the measured versus actual position data. The positioning accuracy can be approximated from the linearity specification by doubling the linearity specification. For example, a 0.02% linearity for a 100 µm stage is a 20 nm maximum deviation. The approximated accuracy error is 2 x 20 nm or 40 nm pk-pk.

4.2 Resolution

Resolution is defined as the smallest detectable mechanical displacement of a piezo nanopositioning stage. Many piezo stage manufacturers will state that the resolution of a piezo actuator is theoretically unlimited because even the smallest change in electric field will cause some mechanical expansion (or contraction) of the piezo stack. Although theoretically true, this fact is largely impractical because all piezo actuators and stages are used with electronics and sensors that produce some amount of noise. The noise in these devices generally rises with increasing measurement sensor bandwidth. As a result, the resolution (or noise) of a piezo nanopositioner is a function of the sensor bandwidth of the feedback device. Aerotech’s piezo amplifiers and feedback electronics have been optimized to provide low noise and high resolution making them suitable for some of the most demanding performance applications.

Aerotech specifies the resolution as a 1 sigma (rms) noise, or jitter, value as measured by an external sensor (either precision capacitance sensor or laser interferometer) at a measurement bandwidth of 1 kHz, unless noted. The stage servo bandwidth is set to approximately 1/3 to 1/5 of the 1st resonant frequency of the piezo nanopositioner because this is generally the highest frequency that the servo bandwidth can be increased to before servo instability occurs. Because the noise is primarily Gaussian, taking six times the 1 sigma value gives an approximation of the pk-pk noise. Unless specified, the measurement point is centered and at a height of approximately 15 mm above the output carriage. In noise critical applications, measuring at a lower servo bandwidth will result in a lower noise (jitter).

Values are specified for open-loop and closed-loop resolution. Open-loop resolution is governed only by the noise in the power electronics whereas closed-loop resolution contains feedback sensor and electronics noise as well as power amplifier noise.

4.3 Repeatability

The repeatability of Aerotech’s QNP piezo nanopositioning stages is specified as a 1 sigma (standard deviation) value calculated from multiple bidirectional full-travel measurements. To obtain an approximate peak-peak value for bidirectional repeatability, multiply the 1 sigma value by 6. For example, a 1 nm value specified as a 1 sigma repeatability will be approximately 6 nm peak-peak.

Unless specified, specifications are measured centered and at a height of approximately 15 mm above the output carriage.

4.4 Stiffness

The stiffness of a piezo actuator or nanopositioner is specified in the direction of travel of the output carriage. The stiffness is a function of the piezo stack, stage flexure and amplification mechanism(s) used in the design. Higher
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stiffness piezo stages allow for higher dynamics in positioning such as faster move and settle times and better dynamic tracking performance.

As mentioned in Section 3.1, most longer-travel (>50 μm) piezo flexure stages use lever amplification to achieve longer travels in a more compact package size. Lever amplification designs cause the stiffness in the direction of travel (inversely proportional to the square of the lever amplification ratio) to be reduced when compared to a directly-coupled design. Also, most lever amplification designs cause the stiffness of the actuator to change depending on location in travel due to the non-linear nature of the amplification gain. For this reason, along with manufacturing and device tolerances, the stiffness of Aerotech’s piezo nanopositioning stages is specified at a nominal value of ±20%.

Aerotech piezo nanopositioning stages are optimized to provide both premium dynamic performance and a compact stage package.

\[ f_n = \left(\frac{1}{2\pi}\right) \sqrt{\frac{k}{m_{\text{eff}}}} \quad (\text{Eq. 14}) \]

estimated as follows:

\[ f_n = (1/2\pi) \cdot \sqrt{k/m_{\text{eff}}} \quad (\text{Eq. 14}) \]

where \( f_n \) is the resonant frequency (Hz), \( k \) is the stiffness of the piezo nanopositioner (N/m) and \( m_{\text{eff}} \) is the effective mass of the stage (kg).

In a very general sense, it is typically the first (lowest) resonant frequency of the positioning system that limits the achievable servo bandwidth. The design of the flexure, supporting mechanics and piezo actuator stiffness govern the location of this resonant frequency. Aerotech has optimized the dynamics of our nanopositioning piezo stages to provide a stiff, high-resonant frequency design in an optimal stage package.

\[ f_n' = \left(\frac{1}{2\pi}\right) \cdot \sqrt{\frac{k}{m_{\text{eff}} + m_{\text{load}}}} \quad (\text{Eq. 15}) \]

Equations 14 and 15 will provide a first-order approximation of resonant frequency in piezo nanopositioning systems. Complex interactions of the dynamics due to damping, nonlinear stiffnesses and mass/inertia effects cause these calculations to provide only an approximation of the resonant frequency. If a more exact value is required for your application or process, please contact us and we will assist in the design and analysis of an engineered solution.

Aerotech specifies the resonant frequency of our piezo nanopositioning stages at a nominal value with a ±20% tolerance along with the given payload (unloaded, 100 grams, etc.).

4.6 Load Ratings

Piezo actuators are ceramic materials and are brittle. As with most ceramics, PZTs have a higher compressive strength than tensile strength. The actuators used in our stage designs are preloaded so as to always maintain a compressive load state during standard operational limits. On our data sheets, we specify push and pull load limits that refer to loading applied in the direction of travel. For some stages, the load rating may be different depending on the direction of the applied load. All Aerotech load ratings are a maximum value. If you require larger load ratings than what is provided in our data sheets, please contact an Aerotech Applications Engineer as we may be able to easily modify or customize a design to meet your exact needs.

4.7 Expected Lifetime

The critical guidance elements in Aerotech piezo actuator flexure stages are sized using FEA and analytical techniques to ensure long, reliable operation. The materials and dimensions chosen for these flexure elements ensure elastic bending and stresses in critical areas well below the endurance limit.

Factors such as humidity, temperature and applied voltage all affect the lifetime and the performance of piezo
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actuators. As discussed in Section 3.7, our actuators are sealed and life-tested to ensure thousands of hours of device life. Based on empirical data developed over years of testing, we can provide lifetime estimates based on the desired move profiles and expected environmental conditions where the piezo nanopositioning system will reside.

5. Amplifier Selection
This section gives a basic overview for selecting a piezo amplifier based on a given piezo actuator and move profile.

Because the displacement of a piezo stage is proportional to the applied voltage, the basic travel is defined by the operating voltage of the amplifier. In our data sheets for open-loop operation, a voltage range is given alongside the open-loop travel. Typically, the closed-loop travel is less than the open-loop travel because closed-loop control usually requires larger voltage margins to achieve equivalent travels (due to hysteresis, dynamic operation, creep, etc.). Although the margins used for closed-loop control are stage and application dependent, it is conservative and safe to assume that closed-loop travel is achieved using the voltage range specified for open-loop control.

Most applications require some form of dynamic operation. Even if the application is positioning a sample or optic at various points in travel and dwelling for long periods of time, the piezo stage will need to move to those positions. At operational frequencies well below the piezo stack’s velocity of the piezo stage). This is different than a typical Lorenz-style servomotor that only draws current during acceleration and deceleration (neglecting losses).

\[ i_{\text{cont}} = \frac{1}{\sqrt{T_2 - T_1}} \int_{t_1}^{T_2} [i(t)]^2 \, dt \quad (\text{Eq. 17}) \]

\[ i_{pk} = \max |i(t)| \quad (\text{Eq. 18}) \]

\[ i_{pk} = \max |i(0)| \quad (\text{Eq. 18}) \]

The current requirements of the desired move profile should be compared against these specifications to determine if the amplifier is capable of sourcing the desired current to the piezo actuator.

The example curve shown in Figure 10 gives the maximum

![Figure 10. Maximum sinusoidal peak-peak voltage of a given amplifier with various piezo stack capacitances.](attachment:image.png)
peak-peak voltage possible for an amplifier, based on the current ratings and frequency of operation for sinusoidal motion of various piezo stack capacitances.

Consider the following additional examples of voltage, power and current calculations for selecting a piezo stage:

Example 1
A 100 μm pk-pk sinusoidal motion at 35 Hz is desired from a stage with a piezo capacitance of 5 μF. The selected amplifier has a semi-bipolar supply of +150 V/-30 V, a 300 mA peak current rating and a 130 mA continuous current rating. Will this amplifier be able to supply enough current to perform this move?

Example 1 Calculations
Assume that to perform the 100 μm pk-pk motion, the full voltage range is used and at mid-travel, the voltage is at the mean of the rail voltages (e.g., 60 V). Therefore:

\[ V(t) = 90 \cdot \sin(2 \cdot \pi \cdot 35 \cdot t) + 60 \]

Recalling that the capacitance can increase by as much as 60% for large signal conditions, the capacitance used for this calculation is assumed to be 5 μF \( \times 1.6 = 8 \mu F \). The current is then calculated as:

\[ i(t) = (2 \cdot \pi \cdot 35) \cdot 90 \cdot 8e^{-6} \cdot \cos(2 \cdot \pi \cdot 35 \cdot t) = 0.158 \cdot \cos(2 \cdot \pi \cdot 35 \cdot t) \]

Therefore, \( i_{pk} = 158 \text{ mA} \) and \( i_{rms} = 112 \text{ mA} \). The voltage and current waveforms are shown in Figure 11.

In this example, the peak and continuous currents are all less than the amplifier rating. Therefore, this amplifier is capable of supplying the necessary current to perform the desired move profile.

Example 2
A move from 0 to 100 μm in 4 ms, dwell for 60 ms, then move back from 100 μm to 0 in 4 ms is the desired output move profile of a stage with a piezo capacitance of 5 μF. The desired amplifier has a semi-bipolar supply of +150 V/-30 V, a 300 mA peak current rating and a 130 mA continuous current rating. Will this amplifier be able to supply enough current to perform this move?

Example 2 Calculations
The same calculations performed in Example 1 are performed using Equations 16, 17 and 18. Again, the capacitance is assumed to increase by approximately 60% due to large signal conditions. The voltage and current waveforms are shown in Figure 12.

In this example, the continuous current is below the rating of the amplifier. However, the peak current exceeds the maximum current rating of the amplifier. Therefore, this amplifier is NOT capable of supplying the necessary current and power to perform the desired move profile.